Matrix Quiz

Put **T** if the statement is **always true**, otherwise put **F** .

O is the zero matrix and I is the identity matrx.

All variables are assumed to be well-defined unless otherwise stated. All equations are linear.

There is no need to prove if the statement is true. Think of a counter-example if it is false.

______**1.** If A is a square matrix and $A^2 = I$, then $A = I$ or $A = -I$.

______**2.** If $AB = O$, then $A = O$ or $B = O$.

______**3.** If A, B, C are square and $ABC = O$, then one of them is O .

______**4.** If $AB = AC$, then $B = C$.

______**5.** If A is non-zero and $AB = AC$, then $B = C$.

______**6.** The square of a non-zero square matrix must be a non-zero matrix

______**7.** If $AB = BA$, then $(A \pm B)^3 = A^3 \pm 3A^2B + 3AB^2 \pm B^3$.

______**8.** An invertible matrix must be square.

______**9.** $(AB)^{-1} = A^{-1}B^{-1}$.

______**10.** If A has a zero row or zero column, then A is not invertible.

______**11.** If A is a square matrix which has no zero rows or zero columns, then A is invertible.

______**12.** Let A, B be invertible matrices of same size. Then AB is also invertible.

______**13.** Let A, B be invertible matrices of same size. Then $A \pm B$ is also invertible.

______**14.** If AB is equal to the identity matrix, then A must be invertible matrix.

______**15.** If A, B are square matrices. If $AB = I$, then $BA = I$. Hence, A is invertible.

______**16.** For square matrix A, $AA^T = I$ if and only if $A^TA = I$

______**17.** If AB is invertible, then BA is invertible.

______**18.** If $A^2 \neq O$, then A is invertible.

______**19.** If A is invertible, then $A^2 \neq O$.

______**20.** If A is a square matrix and $A^2 + 8A - I = O$, then A is invertible.

______**21.** A symmetric matrix must be a square matrix.

______**22.** If A is symmetric, so are A^{-1} (if exists) and A^3.

______**23.** If $B = A^T A$, then 3B is symmetric.

______**24.** If A is symmetric, so is $p(A)$, for any polynomial $p(x)$.

______**25.** $\det(A \pm B) = \det A \pm \det B$.

______**26.** $\det(kA) = k \det A$.

______**27.** Three elementary row operations do not change the determinant of a square matrix.

______**28.** If A is row equivalent to B, then $\det A = \det B$.

______**29.** Let A is row equivalent to B, then $\det A$ and $\det B$ are either both zero or both nonzero.

______**30.** Let A be a square matrix without zero rows and columns. Then A must be row equivalent to the identity matrix of the same size.

______**31.** If A, B are nonzero square matrices and A is row equivalent to B, then both A, B are invertible.

______**32.** If A is an invertible matrix and B is row equivalent to A, then B is also invertible.

______**33.** $(A \pm B)^T = A^T \pm B^T$.

______**34.** $(A B)^T = A^T B^T$.

______**35.** If $m < n$, then the system $A_{m\times n}\, x = O$ always has a nontrivial solution.

______**36.** If A is an $m \times n$ matrix with $m > n$, then $Ax = O$ always has a nontrivial solution.

______**37.** If $\det A \neq 0$, then the system of equation $Ax = O$ has non-trivial solution.

______**38.** Let A be an $n \times n$ matrix. If the equation has a unique solution for a given nonzero $n \times 1$ vector b, then $\det A \neq 0$.

______**39.** Let A be $m \times n$ matrix. If the equation $Ax = b$ has a unique solution for a nonzero vector $m \times 1$, then the homogeneous equation $Ax = O$ has only trivial solution.

______**40.** If $\det A = 0$, then the system of equation $Ax = O$ has no solution.

______**41.** If A, B are square matrices and A is not invertible, then AB is not invertible.

______**42.** If row 2 of a determinant is replaced by row 1 – row 2, the determinant remains the same.

______**43.** In a system of 3 linear equations and 3 variables x, y, z, if the determinants $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$, then the systems of equations has infinite numbers of solutions.

______**44.** $\begin{vmatrix} x & z & y \\ y & x & z \\ z & y & x \end{vmatrix}\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$ can be expressed in the form $A^3 + B^3 + C^3 - 3ABC$, where A, B, C are functions of x, y, z, a, b and c.

Solution

1. **F** , $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $A^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$, $A \neq I$ and $A \neq -I$.

2. **F** , $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ // $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

3. **F** , $A = B = C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $ABC = O$

4. **F** , Choose $A = O$, $B \neq C$

5. **F** , $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, $AB = AC = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $B \neq C$.

6. **F** , $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

7. **T**

8. **T** , A is invertible $\Leftrightarrow$ $AB = BA = I$. If A is $m \times n$ and B is $n \times m$, then $m = n$.

9. **F** , $(AB)^{-1} = B^{-1}A^{-1}$ since $(AB)(B^{-1}A^{-1}) = I$.

10. **T** , $\det A = 0$ and hence not invertible.

11. **F** , $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ has no zero row or zero column and has no inverse.

12. **T** , AB is defined and square. $\det(AB) = \det A \times \det B \neq 0$.

13. **F** , $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $A + B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

14. **F** , $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$, $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

15. **T**

16. **T**

17. **F** , $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$, $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$, $BA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \neq I$

18. **F** , $A = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$, $A^2 \neq O$, $\det A = 0 \Rightarrow$ A^{-1} does not exist.

19. **T**

20. **T** , A is square and $A(A + 8I) = I \Rightarrow$ $A^{-1} = A + 8I$

21. **T**

22. **T** $(A^{-1})^T = (A^T)^{-1}$, $(A^n)^T = (A^T)^n$.

23. **T**

24. **T**

25. **F** , $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ det A = 1, det B = 1, det (A + B) = 0

26. **F** , $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ det A = 1, det (kA) = det $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = k^2 \neq k$

27. **F** , 1. If B is obtained from A by interchanging two rows of A, then det B = - det A

2. If B is obtained from A by multiplying a row in A by k, then det B = k det A

3. If B is obtained from A by changing R_i by $R_i + kR_j$, det B = k det A .

28. **F** , $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then $A \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$, det A = -2 ≠ 1 .

29. **T**

30. **F** , $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then $A \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

31. **F** , $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then $A \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = B$

32. **T**

33. **T**

34. **F** , $(AB)^T = B^T A^T$.

35. **T**

36. **F** , $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ has only trivial solution .

37. **F** , Should be det A = 0 .

38. **T**

39. **T**

40. **F** , $\begin{cases} x + y = 1 \\ 2x + 2y = 2 \end{cases}$ has infinite number of solutions.

41. **T**

42. **F** , the new determinant is the negative of the original determinant.

43. **F** , $\begin{cases} 0x + 0y + 0z = 0 \\ 0x + 0y + 0z = 0 \\ 0x + 0y + 0z = 1 \end{cases}$ has no solution.

44. **T** , $A \equiv ax + bz + cy$, $B \equiv ay + bx + cz$, $C \equiv az + by + cx$.